Optimum Design of Concrete Spread Footing by Computer

By J. P. KOHLI

A computer program written for IBM/360 computer is described that does the optimum design of eccentrically loaded footing as per ACI Code 318-63. The footing may be restricted in dimensions in one or both directions.

The program designs a preliminary sized footing based on least area, calculates its cost in dollars, tries six different sizes close to the first size, compares the cost, and chooses the footing with the least cost. It also calculates the quantities of steel and concrete. Results obtained using the computer program are presented and used to show that the footing designed by computer is the optimum footing.

Keywords: computer programs; costs; footings; optimization; reinforced concrete; structural design.

The design of an eccentrically loaded footing involves a trial and error procedure. In the normal procedure a size of footing is selected depending on the load conditions and allowable soil pressure. The soil pressures are then checked out. If the footing is within allowable range it is designed, otherwise another trial is made. Choosing a good size of footing varies with the experience of the designer and leaves little time for economy considerations. With the availability of computers with large storage capacities, it has become possible to insert the economy factors with design and optimize the footings.

In this paper optimum design of footings will be discussed by:
1. Defining the types of problems that can be solved by the program.
2. Describing assumptions and standard built-in features in the program.
3. Briefly describing the theory and method of analysis used in the program.
4. Describing the computer program.
5. Describing an example problem to show the saving occurred by the use of the program.

Types of Problems

The program has the capability of designing a symmetrical footing with single or double eccentricity. The footing dimensions can be restricted in one or two directions and for a given dimension of footing an unsymmetrical footing can also be designed. The computer design of the footing is the cheapest design for the costs included in the program. The program also calculates the quantities of steel and concrete of individual footings and overall quantities.

ASSUMPTIONS AND STANDARDS

Assumptions
1. No tension is allowed in the footing. Though a certain amount of tension is allowed in the footings, most of the footings designed in normal practice have no tension. If the footing has tension because of being restricted in dimensions it can be analyzed as a special case.
2. Design of the footing is based on ACI 318-63.
3. Footing slab is rigid and is freely supported on elastically isotropic mass, i.e., there is a linear distribution of soil pressure at the footing bottom.

Standards built in the program
1. Minimum thickness of footing is built in as 12 in. (30.5 cm). The thickness increases in increments of 3 in. (7.6 cm).
2. Size of the footing is always a multiple of 6 in. (15.2 cm).
3. Bars used in the design range from #4 to #9 bars. Their maximum and minimum spacings are built in as shown in Table 1.
4. Weight of fill as 100 lb per cu ft (1600 kg/m³).
5. Weight of reinforced concrete as 150 lb per cu ft (2400 kg/m³).
6. f_c = 20,000 psi (140.64 kg/cm²).
7. f_y = 30,000 psi (210.92 kg/cm²) (28 day strength).
8. Cost of concreting (including material) = $35.00/cu yd ($45.80/m³).
9. Cost of formwork = $1.00/sq ft ($11.10/m²).
10. Cost of reinforcing steel = $0.17/lb ($0.38/kg).

The above design standards are based on standard design practice and can vary inside the program depending on designer's requirements.
TABLE 1—MINIMUM AND MAXIMUM SPACING FOR VARIOUS BARS

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Minimum spacing, in. (cm)</th>
<th>Maximum spacing, in. (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.00 (25.4)</td>
<td>12.0 (30.48)</td>
</tr>
<tr>
<td>5</td>
<td>0.80 (20.32)</td>
<td>12.0 (30.48)</td>
</tr>
<tr>
<td>6</td>
<td>0.60 (15.24)</td>
<td>12.0 (30.48)</td>
</tr>
<tr>
<td>7</td>
<td>0.60 (15.24)</td>
<td>12.0 (30.48)</td>
</tr>
<tr>
<td>8</td>
<td>0.60 (15.24)</td>
<td>12.0 (30.48)</td>
</tr>
<tr>
<td>9</td>
<td>0.40 (10.16)</td>
<td>12.0 (30.48)</td>
</tr>
</tbody>
</table>

THEORY AND METHOD OF ANALYSIS

Assuming a straight line stress distribution, the stress at any point \((x, y)\) can be written as:

\[
f(x, y) = f_0 (1 - x/a - y/b) \quad (1)
\]

where

- \(f_0\) = intensity of stress at chosen point of origin
- \(a\) = intercept of neutral axis line on \(x\) axis
- \(b\) = intercept of neutral axis line on \(y\) axis

Assume two edges of footing as two axes and one corner of footing as the origin. If the stress at the origin corresponds to maximum soil pressure, then for sides \(x\) and \(y\) of the footing the opposite corner of the footing should have zero or positive soil pressure for the condition of no tension. There are several possible footings to satisfy two conditions of allowable maximum soil pressure and zero tension, but the one with the least area is chosen as the preliminary size of footing in the program. In choosing the preliminary size, the ratio of small side to large side of the footing is limited to 0.75, for practical considerations.

For a footing, let:

- \(P\) = load on the footing, kips
- \(MX\) = moment about \(x\) axis, ft-kip
- \(MY\) = moment about \(y\) axis, ft-kip
- \(X\) = side of the footing in \(x\) direction, ft
- \(Y\) = side of the footing in \(y\) direction, ft
- \(L = \text{ratio} \, X/Y\)
- \(A\) = area of footing, sq ft
- \(SX = \text{section modulus of footing base along} x\) axis
  \[= X \cdot Y^2/6 = L \cdot Y^3/6 \text{ ft}^3\]
- \(SY = \text{section modulus of footing base along} y\) axis
  \[= Y \cdot X^2/6 = L \cdot X^3/6 \text{ ft}^3\]
- \(Ex = \text{eccentricity in} x\) direction
  \[= MY/P \text{ ft}\]
- \(Ey = \text{eccentricity in} y\) direction
  \[= MX/P \text{ ft}\]
- \(S = \text{maximum allowable soil pressure}, \text{kips per sq ft}\)

\[
a = \frac{(1xy - Yp \cdot Qoy)}{(Qox - Yp \cdot A)} (1xy - Xp \cdot Qox) - \frac{(1ox - Yp \cdot Qoy)}{(Qox - Yp \cdot A)} (1ox - Xp \cdot Qox)
\]

\[
b = \frac{(1xy - Yp \cdot Qoy)}{(Qox - Xp \cdot A)} (1xy - Xp \cdot Qox) - \frac{(1ox - Yp \cdot Qoy)}{(Qox - Xp \cdot A)} (1ox - Xp \cdot Qox)
\]

\[
fo = \text{maximum soil pressure due to load and moments at the chosen point of origin}
\]

\[
S = P/A + MX/SX + MY/SY \quad (\text{for condition of no tension})
\]

\[
S = P/(X \cdot Y) + (6 \cdot P \cdot EY)/(X \cdot Y^2)
+ (6 \cdot P \cdot EX)/(X^2 \cdot Y)
\]

or

\[
S = (P/(X \cdot Y)) [1 + (6 \cdot EY)/Y + (6 \cdot EX)/X]
\]

or

\[
S = (P/(L \cdot Y^2)) [1 + (6 \cdot EY)/Y + (6 \cdot EX)/(L \cdot Y)]
\]

or

\[
S \cdot L \cdot Y^2 = P \cdot (1 + (6 \cdot EY)/Y + (6 \cdot EX)/(L \cdot Y)) = 0
\]

or

\[
Y^2 = (P/(S \cdot L)) \cdot Y - 6 \cdot (P/(S \cdot L)) \cdot (EY + EX/L) = 0
\]

or

\[
f(X, Y) = f_0 (1 - X/a - Y/b) = 0
\]

or

\[
1 - X/a - Y/b = 0
\]

or

\[
X/a + Y/b = 1
\]

or

\[
(L \cdot Y)/a + Y/b = 1
\]

or

\[
L = 1 - Y/b \cdot (a/Y)
\]

or

\[
L = a \cdot (b - Y)/(b \cdot Y)
\]

For a given ratio \(R\), values of \(Y\) can be obtained from Eq. (2). This value of \(Y\) should satisfy Eq. (3); if not, a new value of \(L\) is obtained from Eq. (3) and used in Eq. (2) to obtain a new \(Y\).

The values of \(X, Y\) are obtained for different values of \(L\) from Eq. (2) and (3) for various load conditions and the value with the least area is selected as the preliminary size of the footing. The general equations [Eq. (2) and (3)] for the parameters \(a, b\) of neutral axis are:
where:

- $Q_{ox}$ = first moment of area $A$ about $X$ axis
- $Q_{oy}$ = first moment of area $A$ about $Y$ axis
- $I_{ox}$ = moment of inertia of footing about $X$ axis
- $I_{oy}$ = moment of inertia of footing about $Y$ axis
- $I_{xy}$ = product of inertia of the area about the origin
- $X_p$ = $X$ coordinate of applied eccentric load
- $Y_p$ = $Y$ coordinate of applied eccentric load

However, in the actual program, Eq. (2) is used to determine values of $X$ and $Y$ for values of $L$ ranging from 0.75 to 1.33 and the soil pressures are checked out. The size is increased until no tension requirement is satisfied. Once a footing size is established, moment, reinforcement, shear, bond, etc., are checked. Footing thickness and size are varied until all the requirements are satisfied.

After the footing is designed, the cost of the footing is calculated and footing size is changed (by 6 in.) and a footing redesigned. Six such trials are made within the range of $L$ from 0.75 to 1.33 and the cost of the footings compared. The footing with the least cost is the optimum footing.

**COMPUTER PROGRAM**

The program was written for IBM/360 computer which has a storage capacity of 64,000 locations in core storage. The program is written in FORTRAN IV language and requires 18,478 locations. The input data is simple and consists of five cards per footing and the output is on a paper printer. The program took 1½ months to write but the design time of a footing is reduced from 1 hr to about 5 min. Actual computer time is about 2 min per footing. The detailed program will not be described here; however, the input and output will be described and an example problem will be solved to illustrate the sample output and the optimization of the footing. Defining the following items (Fig. 1), we have:

- $SP$ = allowable soil pressure, kips per sq ft
- $P$ = load on top of the footing, kips
- $AMX$ = moment (other than wind) about $X$ axis in $Y$ direction, ft-kips (Use negative sign for opposite direction)
- $AMY$ = moment about $Y$ axis in $X$ direction, ft-kips (Use negative sign for opposite direction)
- $WMX$ = wind moment about $X$ axis in $Y$ direction, ft-kips (Use negative sign for opposite direction)
- $WMY$ = wind moment about $Y$ axis in $X$ direction, ft-kips (Use negative sign for opposite direction)
- $PDX$ = pier dimension in $X$ direction, in.
- $PDY$ = pier dimension in $Y$ direction, in.
- $D$ = depth of bottom of footing from top of the pier, ft
- $DF$ = depth of fill above the bottom of footing, ft
- $FX_1$ = $FX_2$ = restrictions of dimension in $X$ direction, in.
- $FY_1$ = $FY_2$ = restrictions of dimension in $Y$ direction, in.
- $NB$ = number of footings of same type to be analyzed
- $ND$ = code number to describe if the footing is restricted or not

**Input data**

Card 1 — $NB$ (Use $NB = 99$ to sum up all the quantities and stop the program)
Card 2 — Description of the footing
Card 3 — $SP$
Card 4 — $P$, $AMX$, $AMY$, $WMX$, $WMY$, $PDX$, $PDY$, $DF$
Card 5 — $ND$ (=1 for no restriction; start data for next footing from Card 1; = 0 for restriction)
Card 5A — $FX_1$, $FX_2$, $FY_1$, $FY_2$ (IF $ND = 0$, for no restriction in $X$ or $Y$ direction use these values as 10000.0 Start data for next footing from Card 1)

**Output**

Output is divided into four sheets due to design office requirements.

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Sheet 1 — Input data is reprinted
Sheet 2 — Design calculation results and values of shear, bond, etc., for future design office requirements
Sheet 3 — Quantities (concrete, reinforcing steel, and formwork)
Sheet 4 — Information about footing for the draftsman for sketching the footing

EXAMPLE PROBLEM

A sample problem with the following loading conditions is designed by computer:

Load on the footing = 286.0 kips (129.5 kg)
Load moment about X axis = 143.70 ft-kips (19,850 m-kg)
Load moment about Y axis = 100.00 ft-kips (13,800 m-kg)
Wind moment about X axis = 125.10 ft-kips (17,300 m-kg)
Wind moment about Y axis = 125.10 ft-kips (17,300 m-kg)
Pier dimension = 27 x 27 in. (68.6 cm x 68.6 cm)
Depth of bottom of footing below top of pier = 4.5 ft (1.35 m)
Depth of fill = 4.0 ft (1.22 m)

The same loading conditions were used to design different footings for different side ratios by computer by restricting the dimension of the footings. The cost of each footing was calculated and plotted as ordinates, the ratio of the sides being the abscissa. The curve shown in Fig. 2 shows that the cost of the footing varies as a U curve, with the lowest point of curve being at L = 0.889. For a particular loading condition there is only one value of L for which the cost of footing is least. The computer always designs the footing for this value of L, and therefore the computer design of the footing is the optimum design.

Results obtained by computer for footings of different sizes for the same loading conditions are presented in tabular form. Table 2 presents the footing quantities and Table 4 presents the costs of the footings. Ratio L (x/y) and costs of the footings presented in Table 3 are plotted in Fig. 2 as the optimization curve. The results obtained for the computer design of the footing are shown in Table 4.

**TABLE 2—FOOTING QUANTITIES**

<table>
<thead>
<tr>
<th>Ratio, L (x/y)</th>
<th>Footing size, ft x ft x ft</th>
<th>Reinforcement</th>
<th>Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m x m x m)</td>
<td>X direction</td>
<td>Y direction</td>
</tr>
<tr>
<td>0.5</td>
<td>5x10x20</td>
<td>27-28</td>
<td>23-29</td>
</tr>
<tr>
<td>0.56</td>
<td>9.5x11x11.75</td>
<td>27-28</td>
<td>25-29</td>
</tr>
<tr>
<td>0.625</td>
<td>10.5x10x1.75</td>
<td>27-28</td>
<td>23-29</td>
</tr>
<tr>
<td>0.577</td>
<td>10.5x15.5x1.75</td>
<td>27-27</td>
<td>25-29</td>
</tr>
<tr>
<td>0.733</td>
<td>13.5x13.5x1.75</td>
<td>28-27</td>
<td>21-29</td>
</tr>
<tr>
<td>0.688</td>
<td>15.5x13.5x1.75</td>
<td>27-26</td>
<td>26-28</td>
</tr>
<tr>
<td>0.889</td>
<td>20.5x15.5x1.75</td>
<td>27-28</td>
<td>23-29</td>
</tr>
<tr>
<td>0.966</td>
<td>15.5x13.5x1.75</td>
<td>27-23</td>
<td>23-29</td>
</tr>
<tr>
<td>1.00</td>
<td>15.5x15.5x1.75</td>
<td>27-28</td>
<td>23-29</td>
</tr>
<tr>
<td>1.125</td>
<td>22.5x22.5x1.75</td>
<td>27-28</td>
<td>23-29</td>
</tr>
<tr>
<td>1.26</td>
<td>35.5x35.5x1.75</td>
<td>27-28</td>
<td>25-27</td>
</tr>
</tbody>
</table>

**TABLE 3—FOOTING COST**

<table>
<thead>
<tr>
<th>Ratio, L (x/y)</th>
<th>Concrete $30/cu yd</th>
<th>Steel $0.17/lb</th>
<th>Formwork $2.50/ft</th>
<th>Total cost $ (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>436.45</td>
<td>294.51</td>
<td>130.56</td>
<td>861.56</td>
</tr>
<tr>
<td>0.56</td>
<td>394.30</td>
<td>322.22</td>
<td>117.50</td>
<td>824.02</td>
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<tr>
<td>0.625</td>
<td>381.15</td>
<td>316.69</td>
<td>135.75</td>
<td>708.61</td>
</tr>
<tr>
<td>0.677</td>
<td>387.10</td>
<td>295.33</td>
<td>135.75</td>
<td>705.18</td>
</tr>
<tr>
<td>0.733</td>
<td>397.35</td>
<td>285.88</td>
<td>135.75</td>
<td>703.93</td>
</tr>
<tr>
<td>0.889</td>
<td>396.20</td>
<td>278.68</td>
<td>135.75</td>
<td>702.83</td>
</tr>
<tr>
<td>0.966</td>
<td>396.70</td>
<td>285.96</td>
<td>135.75</td>
<td>702.83</td>
</tr>
<tr>
<td>1.04</td>
<td>396.75</td>
<td>295.34</td>
<td>135.75</td>
<td>702.83</td>
</tr>
<tr>
<td>1.125</td>
<td>396.75</td>
<td>295.34</td>
<td>135.75</td>
<td>702.83</td>
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<tr>
<td>1.26</td>
<td>396.20</td>
<td>279.68</td>
<td>135.75</td>
<td>702.83</td>
</tr>
<tr>
<td>1.36</td>
<td>396.20</td>
<td>279.68</td>
<td>135.75</td>
<td>702.83</td>
</tr>
</tbody>
</table>

Fig. 2—Optimization curve

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First the initial size of the footing is established from Eq. (2) based on the least area for zero soil pressure at one end of the footing and the allowable maximum soil pressure at the other end, within the specified side ratio L limit. Then using this as the starting basis the footing is completely designed. It may be modified in size if necessary to meet the stress requirements or any size restriction requirements specified by the designer. The cost of the footing is then calculated. The cost includes only those items that will make the difference in the cost of the different size footings (overhead, etc., not included), namely, the cost of concrete, reinforcement, and formwork. The differential cost (between the various footing sizes designed) for excavation is neglected as it is negligible. Costs for detailing fabricating bending and placing of reinforcements are included in terms of cost per pound of reinforcement steel. Four more footings are designed such that one of their sides is fixed as (I) 6 in. less (II) 1 ft less (III) 6 in. more (IV) 1 ft more, than that side for the originally designed footing. Costs of all five footings are compared, and the one with the least cost is selected to be the computer designed footing. Using Eq. (2) as the starting basis, it is found that the optimum footing is obtained as one of the five footings.

**CONCLUSIONS**

Computer solution of the footing is the optimum design and it involves savings in computation time as well as in the cost of the structure itself. The program can design the following cases:

1. Symmetrical spread footings with none, single or biaxial bending (the bending may be due to load moments or wind moments)
2. Symmetrical spread footings with restriction in one, or two directions with none, single or biaxial bending
3. Unsymmetrical spread footings with restriction in both directions with none, single or biaxial bending

The input data for the program is small and simple and its output is quite descriptive. A footing can be designed in less than 5 min.

The advantages of a computer solution in a problem of this type are obvious; it saves a lot of computational time for the designer and enables him to utilize it towards the engineering aspects of the problem.

**ACKNOWLEDGMENTS**

The author wishes to acknowledge Donald C. Gregory, Ross D. Johnson, for their valuable suggestions for the footing design; E. L. Smith for his help in the computer program; and the UCC Computer Center for the use of the computer by which these results were obtained.
REFERENCES
1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Detroit, June 1963, 144 pp.

APPENDIX

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>intercept of neutral axis line on X axis</td>
</tr>
<tr>
<td>A</td>
<td>area of the footing</td>
</tr>
<tr>
<td>AMX</td>
<td>moment about X axis</td>
</tr>
<tr>
<td>AMY</td>
<td>moment about Y axis</td>
</tr>
<tr>
<td>b</td>
<td>intercept of neutral axis line on Y axis</td>
</tr>
<tr>
<td>D</td>
<td>depth of the footing bottom below the top of the pier</td>
</tr>
<tr>
<td>DF</td>
<td>depth of fill</td>
</tr>
<tr>
<td>EX</td>
<td>eccentricity in X direction</td>
</tr>
<tr>
<td>EY</td>
<td>eccentricity in Y direction</td>
</tr>
<tr>
<td>f(x,y)</td>
<td>stress at a point (x,y)</td>
</tr>
<tr>
<td>f(X,Y)</td>
<td>stress at a point (X,Y)</td>
</tr>
<tr>
<td>fc</td>
<td>compressive strength of concrete at 28 days</td>
</tr>
<tr>
<td>fo</td>
<td>stress at the origin</td>
</tr>
<tr>
<td>fs</td>
<td>allowable stress in steel</td>
</tr>
<tr>
<td>FX1</td>
<td>restriction on footing in -X direction</td>
</tr>
<tr>
<td>FX2</td>
<td>restriction on footing in +X direction</td>
</tr>
<tr>
<td>FY1</td>
<td>restriction on footing in -Y direction</td>
</tr>
<tr>
<td>FY2</td>
<td>restriction on footing in +Y direction</td>
</tr>
<tr>
<td>Iox</td>
<td>moment of inertia of footing about X axis</td>
</tr>
<tr>
<td>IoY</td>
<td>moment of inertia of footing about Y axis</td>
</tr>
<tr>
<td>Ixy</td>
<td>product of inertia of the area about the origin</td>
</tr>
<tr>
<td>L</td>
<td>ratio of two sides X/Y</td>
</tr>
<tr>
<td>MX</td>
<td>moment about X axis</td>
</tr>
<tr>
<td>MY</td>
<td>moment about Y axis</td>
</tr>
<tr>
<td>NB</td>
<td>number of footings of similar type to be analyzed</td>
</tr>
<tr>
<td>ND</td>
<td>code number for indicating if footing is restricted</td>
</tr>
<tr>
<td>PDX</td>
<td>pier dimension in X direction</td>
</tr>
<tr>
<td>PDY</td>
<td>pier dimension in Y direction</td>
</tr>
<tr>
<td>Qox</td>
<td>first moment of area A about X axis</td>
</tr>
<tr>
<td>Qoy</td>
<td>first moment of area A about Y axis</td>
</tr>
<tr>
<td>S</td>
<td>allowable soil pressure</td>
</tr>
<tr>
<td>SP</td>
<td>allowable soil pressure</td>
</tr>
<tr>
<td>SX</td>
<td>section modulus of footing about X axis</td>
</tr>
<tr>
<td>SY</td>
<td>section modulus of footing about Y axis</td>
</tr>
<tr>
<td>WMX</td>
<td>wind moment about X axis</td>
</tr>
<tr>
<td>WMY</td>
<td>wind moment about Y axis</td>
</tr>
<tr>
<td>X</td>
<td>side of the footing in X direction</td>
</tr>
<tr>
<td>X1</td>
<td>side to the left of center line in X direction</td>
</tr>
<tr>
<td>X2</td>
<td>side to the right of center line in X direction</td>
</tr>
<tr>
<td>Xp</td>
<td>X coordinate of applied eccentric load</td>
</tr>
<tr>
<td>Y</td>
<td>side of the footing in Y direction</td>
</tr>
<tr>
<td>Y1</td>
<td>side to the left of center line in Y direction</td>
</tr>
<tr>
<td>Y2</td>
<td>side to the right of center line in Y direction</td>
</tr>
<tr>
<td>Yp</td>
<td>Y coordinate of applied eccentric load</td>
</tr>
</tbody>
</table>

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