17-10 BRACKETS AND CORBELS

A bracket or corbel is a short member that cantilevers out from a column or wall to support a load. The corbel is generally built monolithically with the column or wall, as shown in Fig. 17-38. The term “corbel” is generally restricted to cantilevers having shear span-to-depth ratios, \( \alpha/d \), less than or equal to 1.

Structural Action

A strut-and-tie model for a corbel supported by a column is shown in Fig. 17-38. Within the corbel itself, the structural action consists of an inclined compression strut, \( A-C \), and a tension tie, \( A-B \). Shears induced in the columns above and below the corbel are resisted by tension in the column bars and ties and by compression forces in struts between the ties.

In tests, [17-21], [17-22] corbels display several typical modes of failure, the most common of which are yielding of the tension tie; failure of the end anchorages of the tension tie, either under the load point or in the column; failure of the compression strut by crushing or shearing; and local failures under the bearing plate. If the tie reinforcement is
hooked downward, as shown in Fig. 17-39a, the concrete outside the hook may split off, causing failure. The tie should be anchored by welding it to a crossbar or plate. Bending the tie bars in a horizontal loop at the outer face of the corbel is also possible, but may be difficult to do because of bends in two directions and may require extra cover. If the corbel is too shallow at the outside end, there is a danger that cracking will extend through the corbel, as shown in Fig. 17-39b. For this reason, ACI Code Section 11.8.2 requires the depth measured at the outside edge of the bearing area must be at least one-half the depth at the face of the column.

Design of Corbels

ACI Code Section 11.8.1 requires corbels having \( a/d \) between 1 and 2 to be designed using strut-and-tie models, where \( a \) is the distance from the load to the face of the column, and \( d \) is the depth of the corbel below the tie, measured at the face of the column. Corbels having \( a/d \) between 0 and 1 may be designed either using strut-and-tie models, or by the closely related traditional ACI design method, which is based in part on the strut-and-tie model and part on shear friction. This procedure was limited to \( a/d \) ratios less than or equal to 1.0 because little test data was available for longer corbels. Regardless of the design method used, the general requirements in ACI Code Sections 11.8.2, 11.8.3.2.1 and 11.8.3.2.2, 11.8.5, 11.8.6, and 11.8.7 must be satisfied.
Two closely related design procedures for corbels will be presented: design using strut-and-tie models, and design according to ACI Code Section 11.8. The strut-and-tie method is a little more versatile than the ACI method, but both give essentially the same results within the range of application of the ACI Code.

EXAMPLE 17-4  Design of a Corbel via a Strut-and-Tie Model

Design a corbel to support the reaction from a precast beam (Fig. 17-38). The end of the beam is 12 in. wide. The column is 16 in. square. The unfactored beam reaction is 60 kips dead load and 39 kips live load. The beam is partially or fully restrained against longitudinal shrinkage. Use $f'_c = 5000$ psi normal-weight concrete and $f_s = 60,000$ psi. Use the load factors and strength-reduction factors from ACI Code Sections 9.2 and 9.3.2.

1. Compute the Factored Load—Corbel. The following load combinations from ACI Code Section 9.2.1 are applicable:

   \[
   U = 1.4(D + F) \quad (ACI \text{ Eq. 9}-1) \\
   U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L, \text{ or } S \text{ or } R) \quad (ACI \text{ Eq. 9}-2)
   \]

   Note that $F$, $T$, $H$, $L$, $S$, and $R$ all are zero.

   \[
   U = 1.4D = 1.4 \times 60 = 84 \text{ kips} \\
   U = 1.2D + 1.6L = 1.2 \times 60 + 1.6 \times 39 = 134 \text{ kips}
   \]

   Thus, the factored vertical load on the corbel is 134 kips.

2. Compute the Distance From the Face of the Column to the Beam Reaction—Corbel.

   Assume a 12-in.-wide bearing plate. From ACI Code Section 10.14.1, the maximum bearing stress is $0.85f'_c = 0.85 \times 5000 = 4250$ psi. Using $\phi = 0.65$ as the strength-reduction factor for bearing, the required width of the bearing plate is

   \[
   \frac{134,000 \text{ lb}}{0.65 \times 4250 \text{ psi} \times 12} = 4.04 \text{ in.}
   \]
ACI Code Section 7.7.1 requires 1.5-in. cover to stirrups or main reinforcement. Try a bearing plate 6 in. by 12 in. by 1.5 in. thick, with the top surface flush with the top of the corbel. This will provide 1.5-in. cover to the principal reinforcement at the top of the corbel.

Assume that the beam extends 7.5 in. past the center of the bearing plate. The design gap between the end of the beam and the face of the column is 1 in. Erection tolerances could make this as large as 2 in. or as small as 0 in. Thus, the beam reaction could act as far as 9.5 in. from the face of the column.

3. **Establish the Depth of the Corbel.** ACI Code Section 11.8 does not give guidance in choosing the size of a corbel. ACI Code Section 11.8.3.2.1 limits the interface shear transfer stress to the smallest of 0.2f', 480 + 0.08f'_c, and 1600 psi. For 5000psi concrete, the second limit governs. We shall use this as guidance, but use a corbel somewhat larger than the minimum. To simplify forming, the corbel will have the same width as the column, b = 16 in. Then

\[
d = \frac{134,000 \text{ lb}}{0.75 \times 880 \text{ psi} \times 16 \text{ in.}} = 12.7 \text{ in.}
\]

where the strength-reduction factor for the strut-and-tie model, \( \phi = 0.75 \), is used.

The smallest corbel that satisfies ACI Code Section 11.9.3.2.1 would have \( b = 16 \text{ in.} \), \( h = 15 \text{ in.} \), and \( d = 15.0 - 1.5 - d_h/2 \) — say, 13 in. For conservatism, try \( b = 16 \text{ in.} \) and \( h = 20 \text{ in.} \); then, assuming No. 8 bars in the tie A-B, \( d = 20 \text{ in.} - (1.5 \text{ in. cover}) - (1/2 \text{ bar diameter}) = 18 \text{ in.} \)

**Try b = 16 in., h = 20 in., d = 18 in.**

4. **Select Design Method—Corbel.** ACI Code Section 11.8.1 allows corbels with \( a/d \) between 1.0 and 2.0 to be designed by using Appendix A. Corbels with \( a/d \) less than 1.0 can be designed by using Appendix A or by using the traditional ACI corbel design method in 11.8. Here, \( a/d = 9.5/18 < 1.0 \), so either design method may be used. We shall use Appendix A.

5. **Select a First Trial Strut-and-Tie Model—Corbel.** ACI Code Section 11.8.3.4 requires corbels to be designed for a shear, \( V_s \), equal to the factored beam reaction and for a factored tensile force, \( N_{tc} \), equal to the actual tension force acting on the corbel, but not less than 0.2\( V_s \). This force represents the forces induced by restrained shrinkage of the structure supported by the corbel [17-23].

**Design the corbel for** \( V_s = 134 \text{ kips} \) and \( N_{tc} = 26.8 \text{ kips} \): (See Fig. 17-38.) The forces \( V_s \) and \( N_{tc} \) can be resolved into an inclined force that intercepts the centroid of the tie reinforcement at node A, which is located at 9.5 + 0.2 \times (1.5 + 0.5) \text{ in.} = 9.9 \text{ in.}, \text{ from the face of the column.}

Figure 17-38 shows the final strut-and-tie model. Assuming (a) that the column reinforcement is No. 9 bars with No. 3 ties and (b) that the nodes B and D are in the plane of the column reinforcement located at 1.5 in. cover + 0.375 in. ties + 1.270/2 = 2.5 in. from the right-hand face of the column, gives the distance from A to B as 9.9 in. + 16 in. - 2.5 in. = 23.4 in.

**Locate node C:** Node C is located at a distance \( a/2 \) from the left side of the column, where \( a \) is the depth of the stress block resisting the force in the strut C-E. ACI Code Section A.5.2 gives the effective compression strength of the nodal zone as

\[
f_{ce} = 0.85 \beta_n f' \text{c} \quad \text{(17-7b)}
\]

(ACI Eq. A-8)

The node at C anchors three compression struts and a tension tie, to be discussed later. From ACI Code Section A.5.2, a nodal zone anchoring one tie has \( \beta_n = 0.80 \), so that

\[
f_{ce} = 0.85 \times 0.80 \times 5000 \text{ psi} = 3400 \text{ psi}
\]

and

\[
a = \frac{P_{u,C-E} \text{kips} \times 1000 \text{ lb/kip}}{0.75 \times 3400 \text{ psi} \times 16 \text{ in.}} = 0.0245P_{u,C-E} \text{ in.}
\]

where \( P_{u,C-E} \) is in kips and the 0.75 is the \( \phi \) factor for strut-and-tie models from ACI Code Section 9.3.2.6. Summing moments about \( D \) gives

\[
\Sigma M_D = 0 = 134 \text{ kips} \times 23.4 \text{ in.} + 26.8 \text{ kips} \times 18 \text{ in.} - P_{u,C-E} \times (13.5 \text{ in.} - a/2)
\]

For \( a = 0.0245P_{u,C-E} \), this becomes

\[
0 = 295,000 - 1100P_{u,C-E} + (P_{u,C-E})^2
\]
Thus, \( P_{n,C-E} = 463 \text{ kips} \) and \( a = 0.0245 \times 463 = 113 \text{ in.} \) Node C is \( 11.3/2 = 5.65 \text{ in.} \) from the left edge of the column adjacent to C, and C–D is 13.5 in. \(-\) 5.65 in. = 7.85 in.

The nodal zone at C takes up 11.35 in. out of the 13.5 in. width between the compression face of the column and node D. For comparison, the rectangular stress block in a beam would have a depth of about 0.3d. This indicates that the column is too small. We shall increase the width of the column to 18 in. The distance from A to B increases to 25.4 in.

6. Recompute \( a \) for the New Depth of Column—Second Model—Corbel. Summing moments about D gives

\[
\Sigma M_D = 0 = 134 \times 25.4 + 26.8 \times 18 - P_{n,C-E} \times (15.5 - a/2)
\]

\[
0 = 317,000 - 1265 P_{n,C-E} + (P_{n,C-E})^2
\]

which gives \( P_{n,C-E} = 345 \text{ kips} \). Thus, \( P_{n,C-E} \) drops from 464 kips to 345 kips, corresponding to \( a = 8.45 \text{ in.} \) and \( a/2 = 4.23 \text{ in.} \). The distance C–D is 15.5 in. \(-\) 4.23 in. = 11.3 in.

7. Solve for Forces in the Struts and Ties—Second Model—Corbel.

Node A: Figure 17-40a shows the forces acting at node A.

*Strut A–C* has a horizontal projection of 25.4 \(-\) 11.3 = 14.1 in. and a vertical projection of 18 in. It supports a factored vertical force component of 134 kips, a horizontal force component of

\[
\frac{14.1}{18} \times 134 \text{ kips} = 105 \text{ kips}
\]

and an axial force of \( \sqrt{134^2 + 105^2} = 170 \text{ kips compression} \). The angle between A–C and the tie A–B is \( \arctan(18/14.1) = 51.9^\circ \).

*Tie A–B:* Summing horizontal forces at node A gives \( P_{n,A-B} = 105 + 26.8 = 132 \text{ kips tension} \).

Node B: Figure 17-40b shows the forces acting at node B.

*Strut B–C:* Member B–C has a horizontal projection of 18 in. and a horizontal projection of 15.5 in. \(-\) \( a/2 \), where \( a/2 = 4.23 \text{ in.} \), making the horizontal projection of B–C 11.3 in. Summing horizontal forces at joint B gives the horizontal component of the force in B–C as 132 kips. The vertical force component in B–C is

\[
\frac{18}{11.3} \times 132 = 210 \text{ kips}
\]

and the axial force is 248 kips.

The angle between B–C and tie B–D is \( \arctan(11.23/18) = 32.1^\circ \).

*Tie B–D:* Summing vertical forces at node B gives \( P_{n,B-D} = 210 \text{ kips tension} \).

Node C: Figure 17-40c shows the forces acting at node C. The sum of vertical forces is 134 kips down \(+\) 210 kips down \(+\) 344 kips up \(= 0 \). This should total zero—OK. Summing horizontal forces gives a tension of 26.8 kips in tie C–D. The angle between strut A–C and tie C–D is 51.9° and the angle between strut B–C and tie C–D is 57.9°.

Node D: Figure 17-40d shows the forces at node D. An inclined strut D–E is required in the column for equilibrium. A tension tie D–F is also required.


*Tie A–B:*

\[
A_t = \frac{132 \text{ kip}}{0.75 \times 60 \text{ kip}} = 2.93 \text{ in.}^2
\]

Possible choices are (a) four No. 8 bars, \( A_t = 3.16 \text{ in.}^2 \), which will fit into a width of 11.5 in. and have a basic hook-development length of 17 in., and (b) five No. 7 bars, \( A_t = 3.00 \text{ in.}^2 \), which will fit into a width of 13 in. and have a basic hook-development length of 14.8 in. Use five No. 7 bars for tie A–B. Hook these with the vertical tails of the hooks in the plane of the right-hand layer of column steel with a cover of 1.5 \(+\) 0.5 in. from the right-hand side of the column.
Fig. 17-40  
Calculation of forces in struts and ties—Corbel—Example 17-4.

**Tie B–D:**

\[
A_t = \frac{210 \text{ kips}}{0.75 \times 60 \text{ ksi}} = 4.67 \text{ in.}^2
\]

The reinforcement for tie B–D will be selected by considering the statics and resistance of the column as a whole. The longitudinal column reinforcement will be sized to provide the required reinforcement for this tie. It may be necessary to enlarge the column further.

**Tie C–D:**

\[
A_t = \frac{26.8 \text{ kips}}{0.75 \times 60 \text{ ksi}} = 0.60 \text{ in.}^2
\]

Use two No. 4 closed column ties, \( A_t = 0.80 \text{ in.}^2 \).

9. **Compute \( f_s \) and the Widths of the Struts—Third Model—Corbel.** These calculations are summarized in Table 17-8.
TABLE 17-8  Widths of Struts and Ties—Corbel—Example 17-4

<table>
<thead>
<tr>
<th>Member</th>
<th>Axial Force</th>
<th>$f_{ce}$ for Strut</th>
<th>$f_{ce}$ for Node</th>
<th>$w_s$ in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A–C at A</td>
<td>170</td>
<td>0.85 × 0.75 × 5000 = 3190</td>
<td>0.85 × 0.8 × 5000 = 3400</td>
<td>4.44</td>
</tr>
<tr>
<td>A–C at C</td>
<td>170</td>
<td>3190</td>
<td>3400</td>
<td>3.33</td>
</tr>
<tr>
<td>B–C at B</td>
<td>249</td>
<td>3490</td>
<td>0.85 × 0.60 × 5000 = 2550</td>
<td>6.10</td>
</tr>
<tr>
<td>B–C at C</td>
<td>249</td>
<td>3190</td>
<td>3400</td>
<td>4.88</td>
</tr>
<tr>
<td>C–E at C</td>
<td>345</td>
<td>4250</td>
<td>3190</td>
<td>6.76</td>
</tr>
</tbody>
</table>

**NOTE:** Two rows are provided for each strut, to allow different values of $f_{ce}$ at the two ends of a strut. Also, use an effective corbel width of 12 in. under the bearing plate at node A. At all other nodes use a width of 16 in.

10. **Draw the Strut-and-Tie Model to Scale—Third Model—Corbel.** This is done to see whether the struts fit within the space available. Figure 17-38 shows that they do with very little extra space.

11. **Provide reinforcement to confine struts—Corbel.** When $f_{ce}$ was computed for the struts, they were all assumed to be bottle-shaped. Such struts must have transverse reinforcement satisfying ACI Code Section A.3.3 or A.3.3.1. These sections allow two methods of calculating the amount required.

**Strut A–B:** Using, ACI Code Section A.3.3.1 requires that the strut be crossed by steel satisfying

$$\sum \frac{A_{sj}}{b_s} \sin \gamma_i \geq 0.003$$

where $A_{sj}$ is the area of transverse steel at an angle $\gamma_i$ to the axis of the strut, which cannot be taken as less than 40°. The computed angle is 51.9°. Use No. 4 closed stirrups at $s_i = 4$ in., spread over the length and width of strut A–C (and B–C)

$$\sum \frac{A_{sj}}{b_s} \sin \gamma_i = \frac{2 \times 0.2}{16 \times 4} \sin 51.9° = 0.00492$$

This exceeds the required 0.003. **Use four No. 4 two-legged closed stirrups at 4 in. o.c.** Place the first one at 4 in below the centroid of tie A–B.

12. **Satisfy the Detailing Requirements—Third Model—Corbel.** ACI Code Section 11.8 presents a number of detailing requirements for corbels.

11.8.2—The depth under the outer edge of the bearing plate shall not be less than half the height at the face of the column—OK. This requirement was introduced to prevent failures like the one shown in Fig. 17-39b.

11.8.6—Probably the most important detailing requirement is that the tie A–B be anchored for the tension tie force at the front face of the corbel (at A). Because the tie in the truss in Fig. 17-38 is assumed to be stressed to $f_j$ in tension over the whole distance from the loading plate to the column, it must be anchored outside the loading plate for that tension. This is done in one of several ways. It can be anchored

(a) Welding the bars making up the tie to a transverse angle or bar, which may also serve as a bearing plate.
(b) Welding the bars making up the tie to a transverse bar of the same diameter as the bars making up the tie.
(c) Bending the bar in a horizontal loop.

Although the tension tie could also be anchored by bending the bars in a vertical bend, as illustrated in Fig. 17-39a, this is discouraged, because failures have occurred when this detailing was used, as shown in that figure. We shall assume that the (6 × 12 × 1.5)-in. bearing plate welded across the ends of the five No. 7 bars, as chosen in step 8, will anchor the tie. Because welding is required, the five No. 7 bars must be specified as Grade-60W steel. Alternatively, a steel angle could be used to anchor tie A–B at A, as shown in Fig. 17-38. The plate is easier to weld, and easier to concrete under, but the angle provides resistance to damage to the outer edge of the bearing area.
11.8.7—requires that the bearing area of the load either

(a) not project beyond the start of the bend of the top tie bar, if it is anchored at node A by a hook, or

(b) not project past the interior face of the transverse anchor bar, if that detail is used.

Welding the $6 \times 4 \times \frac{3}{8} \times 12$-in. bearing angle across the five No. 7 bars with the center of the top of the angle under the center of the reaction bearing plate will anchor the tie.

13. Check the Moments in the Column—Third Model—Corbel. The loads on the corbel cause a moment of

$$134 \times (9.9 + 18/2) = 2530 \text{ kip-in.}$$

about the center of the column on a line joining nodes $A$ and $B$. This moment should be divided between the columns above and below the joint in proportion to the stiffnesses of each of these columns. The strut-and-tie model in Fig. 17-41 gives a more complete idea of the corbel action.
Design of Corbels by ACI Code Method

ACI Code Section 11.8 presents a design procedure for brackets and corbels. It is based in part on the strut-and-tie truss model and in part on shear friction. The design procedure is limited to \(ad\) ratios of 1.0 or less. At the time it was included in the code there was little test data for longer brackets.

In the ACI design method, the section at the face of the support is designed to resist the shear \(V_c\), the horizontal tensile force \(N_{ax}\), and a moment of \([V_c a + N_{ax} (h - d)]\), where the moment has been calculated relative to the tension steel at the face of the column in Fig. 17-38. The maximum shear strength, \(V_c\), shall not be taken greater than the smallest of \(0.2 f'_c b_d d\), \((480 + 0.08 f'_c) b_w d\), and \(1600 b_w d\) lb, for normal-weight concrete.

In design, the size of the corbel is selected so that \(V_c \leq \phi V_{c1}\), based on the maximum shear strength. If a high value of \(V_c\) is used, cracking at service loads may lead to serviceability problems. The designer then calculates the following:

1. The area, \(A_{sy}\), of shear-friction steel required is,
   \[
   V_n = A_{sy} f_y \mu
   \]
   (17-17)
   (ACI Eq. 11-25)

2. The area, \(A_f\), of flexural reinforcement required to support a moment of \([V_c a + N_{ax} (h - d)]\), based on ACI Code Chapter 10 (Chapter 5 of this book).

3. The area, \(A_{ax}\), of direct-tension reinforcement required to resist the tension force \(N_{ax}\), where
   \[
   \phi A_{ax} f_y \geq N_{ax}
   \]
   (17-18)

In all these calculations, \(\phi\) is taken equal to the value for shear, 0.75, which is also the value for strut-and-tie models.

The resulting area of tensile steel, \(A_x\), and the placement of the reinforcement within the corbel are specified in ACI Code Sections 11.8.3.5 and 11.8.4. In the corbel tests reported in [17-21], [17-22], the best behavior was obtained in corbels that had some horizontal stirrups in addition to the tension tie shown in Fig. 17-38. Accordingly, ACI Code Sections 11.8.3.5 and 11.8.4 requires that two reinforcement patterns be considered and the one giving the greater area, \(A_x\), be used

1. A tension tie having area \(A_x = A_f + A_{ax}\) plus horizontal stirrups having area \(A_{sy}/2\)

2. A tension tie having area \(A_x = (2A_{sy}/3) + A_{ax}\) plus horizontal stirrups having area \(A_{sy}/3\).

The horizontal stirrups are to be placed within \(\frac{2}{3} d\) below the tension tie.

**EXAMPLE 17-5** Design of a Corbel—Traditional ACI Code Method

Design a corbel to transfer a precast-beam reaction to a supporting column. The factored shear to be transferred is 134 kips. The column is 16 in. square. The beam being supported is restrained against longitudinal shrinkage. Use \(f'_c = 5000\) psi normal-weight concrete and \(f'_s = 60,000\) psi. Use ACI Code Sections 9.2 and 9.3.

1. Compute the Distance, \(a\), From the Column to \(V_c\). Assume a 12-in.-wide bearing plate.

   From ACI Code Section 10.14, the allowable bearing stress is
   \[
   \phi 0.85 f'_c = 0.65 \times 0.85 \times 5 \text{ ksi} = 2.76 \text{ ksi}
   \]
   The required width of the bearing plate is
   \[
   \frac{134 \text{ kips}}{2.76 \times 12 \text{ in.}} = 4.05 \text{ in.}
   \]
Use a 12-in. × 6-in. bearing plate. Assume that the beam overhangs the center of the bearing plate by 7.5 in., that a 1-in. gap is left between the end of the beam and the face of the column, and that the distance $a$ is assumed to be 9.5 in.

2. **Compute the Minimum Depth, $d$.** Base this calculation on ACI Code Section 11.8.3.2.1:

$$
\phi V_e \geq V_u
$$

$V_u$ is limited to the smallest of $0.2f'_c b_w d$, $(480 + 0.08f'_c) b_n d$, and $1600 b_n d$. For 5000-psi concrete, the second equation governs. Thus,

$$
\text{minimum } d = \frac{V_u}{\phi \times 880 b_w} = \frac{134,000 \text{ lb}}{0.75 \times 880 \times 16} = 12.7 \text{ in.}
$$

Hence, the smallest corbel we could use is a corbel with $b = 16$ in., $h = 15$ in., and $d = 13$ in. For conservatism, we shall use $h = 20$ in. and $d = 20$ in. − (1 ½ in. cover + ½ bar diameter), which equals 18 in. The corbel will be the same width as the column (16 in. wide), to simplify forming.

3. **Compute the Forces on the Corbel.** The factored shear is 134 kips. Because the beam is restrained against shrinkage, we shall assume the normal force to be (ACI Code Section 11.8.3.4)

$$
N_{ac} = 0.2V_u = 26.8 \text{ kips}
$$

The factored moment is

$$
M_u = V_u a + N_{ac} (h - d) = 134 \text{ kips} \times 9.5 \text{ in.} + 26.8 \text{ kips} (20 \text{ in.} - 18 \text{ in.}) = 1330 \text{ kip-in.}
$$

4. **Compute the Shear Friction Steel, $A_{sf}$** From Eq. (17-17),

$$
\phi V_e \geq V_u
$$

$$
A_{sf} = \frac{V_u}{\mu f_y} = \frac{V_u}{\phi \mu f_y}
$$

where $\mu = 1.4\lambda$ for a shear plane through monolithic concrete and $\lambda = 1.0$ for normal-weight concrete. Therefore,

$$
A_{sf} = \frac{134 \text{ kips}}{0.75(1.4 \times 1.0)60 \text{ ksi}} = 2.13 \text{ in.}^2
$$

5. **Compute the Flexural Reinforcement, $A_f$** $A_f$ is computed from Eq. (5-16) (with $A_s$ replaced by $A_f$):

$$
M_u = \phi A_f f_y \left( d - \frac{a}{2} \right)
$$

Here, $\phi = 0.75$ (ACI Code Section 11.8.3.1) and

$$
a = \frac{A_f f_y}{0.85 f'_c b}
$$

As a first trial, we shall assume that $(d - a/2) = 0.9d$. Thus,

$$
A_f \geq \frac{M_u}{\phi f_y (0.9d)} \geq \frac{1330 \text{ k-in.}}{0.75 \times 60 \times 0.9 \times 18} = 1.82 \text{ in.}^2
$$
Because this is based on a guess for \((d - a/2)\), we shall compute \(a\) and recompute \(A_f\):

\[
a = \frac{1.82 \times 60}{0.85 \times 5 \times 16} = 1.61 \text{ in.}
\]

\[
A_f \geq \frac{1330 \text{ kip-in.}}{0.75 \times 60 \text{ ksi} \ (18 - 1.61/2) \text{ in.}}
\]

\[
\geq 1.72 \text{ in.}^2
\]

Therefore, use \(A_f = 1.72 \text{ in.}^2\).

6. **Compute the Reinforcement, \(A_n\), for Direct Tension.** From ACI Code Section 11.8.3.4,

\[
A_n = \frac{N_{nc}}{\phi f_y} = \frac{26.8 \text{ kips}}{0.75 \times 60 \text{ ksi}}
\]

\[
= 0.60 \text{ in.}^2
\]

7. **Compute the Area of the Tension-Tie Reinforcement, \(A_{nc}\).** From ACI Code Section 11.8.3.5, \(A_{nc}\) shall be the larger of

\[
(A_f + A_n) = 1.72 + 0.60 = 2.32 \text{ in.}^2, \text{ or}
\]

\[
\left(\frac{2A_{eff}}{3} + A_n\right) = 1.42 + 0.60 = 2.02 \text{ in.}^2
\]

Minimum \(A_{nc}\) (ACI Code Section 11.8.5):

\[
A_{nc,(min)} = \frac{0.04f_y^2}{f_y} b_{ad}d = 0.96 \text{ in.}^2
\]

Therefore, \(A_{nc} = 2.32 \text{ in.}^2\). Try three No. 8 bars, giving \(A_{nc} = 2.37 \text{ in.}^2\).

8. **Compute the area of horizontal stirrups.**

\[
0.5(A_{nc} - A_n) = 2.32 - 0.60 = 0.86 \text{ in.}^2
\]

Select three No. 4 double-leg stirrups, area = 1.20 in.\(^2\); ACI Code Section 11.8.4 requires that these be placed within \((2/3)d\), measured from the tension tie.

9. **Establish the anchorage of the tension tie into the column.** The column is 16 in. square. Try a 90° standard hook. From ACI Code Section 12.5.1,

\[
\ell_{sh} = \frac{0.02\psi_s f_y}{A\sqrt{f'_e}} = \frac{0.02 \times 1 \times 60,000}{1 \times \sqrt{5000}} = 17.0
\]

measured from the face of the column. Because these bars will be placed inside the column bars, the modification factor (0.7) in ACI Code Section 12.5.3(a) will apply, so \(\ell_{sh} (\text{mod.}) = 17.0 \times 0.7 = 11.9\) in. Therefore, use three No. 8 bars hooked into the column. The hooks are inside the column cage.

10. **Establish the anchorage of the outer end of the bars.** The outer end of the bars must be anchored to develop \(A_{nc} f_y\). This can be done by welding the bars to a transverse plate, angle, or bar. If the horizontal force, \(N_{hc}\), is required for equilibrium of the structure, some direct connection would be required from the beam base plate to the tension tie. This is not the case here, and a welded cross-angle will be provided, as shown in Fig. 17-38.

11. **Consider all other details.** To prevent cracks similar to those shown in Fig. 17-39b, ACI Code Section 11.8.2 requires that the depth at the outside edge of the bearing area be at least 0.5d. ACI Code Section 11.8.7 requires that the anchorage of the tension tie be outside the bearing area. Finally, two No. 4 bars are provided to anchor the front ends of the stirrups. All of these aspects are satisfied in the final corbel layout which is similar to Fig. 17-38.
Comparison of the Strut-and-Tie Method and the ACI Method for Corbel Design

The strut-and-tie method required more steel in the tension tie and less confining reinforcement than the ACI method. The strut-and-tie method explicitly considered the effect of the corbel on the forces in the column. The strut-and-tie method could also be used for corbels that have $a/d$ greater than the limit of 1.0 given in ACI Code Section 11.8.1. For $a/d > 1$, the confining stirrups would be more efficient in restraining the splitting of the strut if they were vertical.