Cálculo Numérico de Figuras Planas y Zonas Comprimidas de Concreto

CLAVES: cálculo numérico; computadores; concreto armado; geometría; inercia; programación; resistencia de materiales; hormigón; ordenadores.

RESUMEN

Este artículo presenta un procedimiento simple, general, directo y sistemático para calcular numéricamente el baricentro y el tensor de inercia de cualquier figura plana formada por un contorno poligonal, a través de fórmulas que los refieren directamente a los ejes elegidos por el usuario, por medio de una simple tabulación programable en cualquier computador electrónico de mesa de poca capacidad. Se incluye un algoritmo para definir subfiguras en banda resultantes de un contorno dado, tales como las que se requieren generar al analizar las zonas comprimidas de una sección de concreto armado sometida a flexocompresión. Este suministra un corto subprograma aplicable en la programación tanto del cálculo de volúmenes, flexocompresión y pandeo, como en la de dibujos con el delineador electrónico.

NUMERICAL COMPUTATION OF PLANE FIGURES AND CONCRETE COMPRESSED ZONES

KEY WORDS: bending; **computer programming; concrete (reinforced); engineering mechanics; geometry; inertia; numerical analysis.**

ABSTRACT

This paper presents a simple, general, direct and systematic procedure for computing numerically the centroid and tensor of inertia of plane polygonal figures. The formulae allow the computation of the tensor of inertia of any polygonal contour directly referred to the user axes, either by straightforward tabulation or by a small electronic desk computer. An algorithm for defining band subfigures, as required in reinforced concrete bending problems, generated from a previously existing polygon is included. This provides a simple subprogram to be applied in volume, bending, buckling and plotter programming.

NUMERICAL COMPUTATION OF PLANE FIGURES AND CONCRETE COMPRESSED ZONES

By Joaquín Marín[1]

1. INTRODUCTION

This paper intends to divulge a simple, general and systematic procedure for computing numerically the area, centroid and tensor of inertia of plane polygonal figures which are referred to a set of arbitrarily chosen orthogonal axes. The method and formulae presented had to be fully derived by the author during the development of a computer program for revising reinforced concrete short columns of any shape which are subjected to combined axial load and biaxial bending. This was necessary since no algorithm capable of solving such a frequent engineering problem was found in the author's available literature.

The numerical technique is based on the well known trapezoid superposition traditional in discrete mathematics. The fundamental problem of which trapezoid properties should be added or substracted is solved analytically by a simple rule of numbering vertices. Then a set of easy, useful, straightforward formulae are found. The simplicity of these formulae is such that the author wonders why they are not found as part of the current numerical tools of every engineer since his first course in mechanics.

In addition, a simple general algorithm for programming the computation of mechanical properties of straight sections of a plane figure is introduced. This algorithm has been found to be more practical and accurate in studying the above mentioned problem than those occasionally found in the current literature.

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2. DEFINITIONS AND NOTATION

Positive Sense:	In numbering sequentially the vertices of a polygonal figure, the resulting rotation defined usually in mathematics as positive, i.e., counter-clock wise.
Positive Axes:	A pair of orthogonal coödinate axes, where the x axis should rotate in a positive sense to meet the y axis.
Subfigure:	One part of a figure which has been divided by a straight line that crosses it.
Band Subfigure:	One zone of a figure limited by two parallel straight lines.
Band Polygon:	Polygon describing analytically the geometric information of a band subfigure.
a_i	Expression defined in formula (6).
A	Area.
A_k	Auxiliar variable, k = 1,2
INT	Number of intersections between the original figure and the subfigure limiting straight lines.
IXX, IYY, IXY	Components of the plane tensor of inertia.
MAX, MAY	Moments of area referred to the x and y axes, respectively.
N	Number of sides or different vertices of the figure.
$\overline{N}$	Number of sides or vertices of the resulting subfigure.
x_i , y_i	Coördinates of the points forming the plane figure referred to the user positive axes. i = 1,N
x'_i , y'_i	Coördinates of the vertices of the figure rotated α.

$\overline{x_i'}$, $\overline{y_i'}$	Coördinates of the points defining the resulting subfigure polygon rotated the angle α . $i = \overline{1,N}$
x'	Divisor direction. Axis whose direction orientates the straight lines which the figure is to be divided with.
XC , YC	Coördinates of the centroid.
y'_T , y'_B	Top and bottom ordinates, referred to the divisor axis x', that determine the two limiting straight parallel lines of a band subfigure.
α	Angle formed by the user x axis and the divisor direction x'.

3. METHOD AND FORMULAE

In order to compute systematically the area, moments of area, centroid, and the tensor of inertia of a plane figure referred to a pair of positive coördinate orthogonal axes arbitrarily chosen by the user, the following method is based on the three assumptions shown in Figure 1:

a) The figure is defined by a closed polygon of N straight sides whose vertices are determined by the N pairs of known coördinates x_i , y_i.

b) The contour of the figure is simply connected; i.e., it is possible to travel over all of it and return to the starting point. Obviously this is not a limitation since any figure, for the purpose of these computations, may easily and precisely be converted into a simply connected one by a proper fictitious sectioning.

c) The vertices of the closed simply connected polygon have to be ordered in sequence.

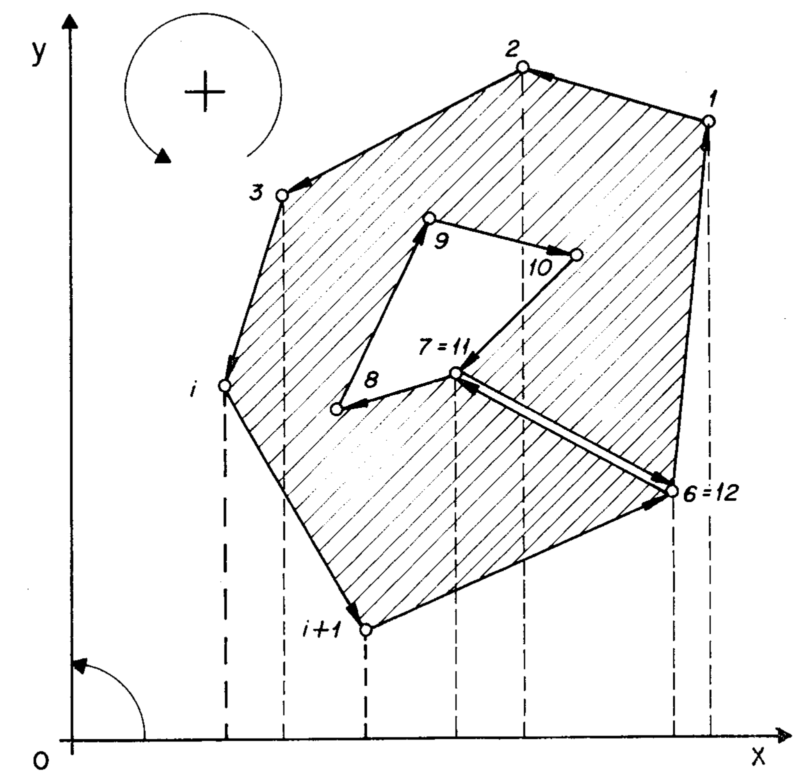

FIG. 1.—ASSUMPTIONS AND CONVENTIONS.

By satisfying assumption (a) the numerical computation of the mechanical properties of a plane figure can be performed by superposing the properties of the rectangular trapezoids which are formed when the extreme points of each side are projected on one of the coördinate axes.

By meeting assumption (c), the fundamental question about which of these individual trapezoid properties ought to be added or substracted is automatically and analytically solved, regardless of how intricate the contour might be. For example, if a positive sense of numbering is selected, as shown in Fig. 1, and x-areas are required, it is evident that x-areas of trapezoids whose sloping sides advance in the abscisae direction have to be substracted, no matter how laberynthine the figure is. This simple convention of sequencing makes the computations completely systematic.

For such a trapezoid the following formulae are readily derived, based on the positive sense of numbering vertices previously defined. Otherwise the formulae should be multiplied by minus one

Moments of Area

$$MAX_{i,i+1} = \frac{1}{6}(x_i - x_{i+1})(y_i^2 + y_i y_{i+1} + y_{i+1}^2) \qquad (1)$$

$$MAY_{i,i+1} = \frac{1}{6}(y_{i+1} - y_i)(x_i^2 + x_i x_{i+1} + x_{i+1}^2) \qquad (2)$$

Moments and Product of Inertia

$$IXX_{i,i+1} = \frac{1}{12}(x_i - x_{i+1})(y_i^3 + y_i^2 y_{i+1} + y_i y_{i+1}^2 + y_{i+1}^3) \qquad (3)$$

$$IYY_{i,i+1} = \frac{1}{12}(y_{i+1} - y_i)(x_i^3 + x_i^2 x_{i+1} + x_i x_{i+1}^2 + x_{i+1}^3) \qquad (4)$$

$$IXY_{i,i+1} = \frac{1}{24}(x_i - x_{i+1})\left[(x_i + x_{i+1})(y_i + y_{i+1})^2 + 2(x_i y_i^2 + x_{i+1} y_{i+1}^2)\right] \qquad (5)$$

Then the formulae are extended to the whole contour and optimized, both in operations and memory registers. The most convenient set is that given in Reference 1, the only pertinent information found on this topic.

Let: $$a_i = x_i y_{i+1} - x_{i+1} y_i \qquad (6)$$

Area, Moments of Area and Centroid

$$A = \frac{1}{2} \Sigma a_i \qquad (7)$$

If the area becomes null or negative, the figure has been incorrectly sequenced. However, a positive result in and of itself does not guarantee that a topological mistake has not been made.

$$MAX = \frac{1}{6} \Sigma a_i (y_i + y_{i+1}) \qquad MAY = \frac{1}{6} \Sigma a_i (x_i + x_{i+1}) \qquad (8)$$

$$YC = MAX / A \qquad XC = MAY / A \qquad (9)$$

Moments and Product of Inertia

$$IXX = \frac{1}{12} \Sigma a_i \left[(y_i + y_{i+1})^2 - y_i y_{i+1}\right] \qquad (10)$$

$$IYY = \frac{1}{12} \Sigma a_i \left[(x_i + x_{i+1})^2 - x_i x_{i+1}\right] \qquad (11)$$

$$IXY = \frac{1}{12} \Sigma a_i \left[(x_i + x_{i+1})(y_i + y_{i+1}) - \frac{1}{2}(x_i y_{i+1} + x_{i+1} y_i)\right] \qquad (12)$$

Summations should be extended to the N sides of the polygon; i.e., to its N vertices, the N + 1 vertex being the first. It is to be remembered that the formulae evaluate the geometric and mechanical characteristics referred to the coördinate axes arbitrarily selected by the user.

4. SUBFIGURE POLYGONS AND ITS APPLICATION TO CONCRETE COMPRESSED ZONES

Very frequently engineers face the problem of calculating geometric and mechanical properties of plane figures generated from a previously existing one by crossing it with straight lines. The computation of volumes in mathematics and the study of the strength of members subjected to combined bending and axial load are perhaps the most relevant examples.

Nevertheless, to the author's knowledge, no general systematic straightforward numerical technique is presently available.

In order to apply the former formulae to any subfigure the following problem must be solved: Consider an L shape contour, as a simple example, divided by the straight line x′ shown in Fig. 2 and suppose that positive ordinates y′ define the zone of interest. Thus, it is seen that two separate parts of the original shape appear in the subfigure:

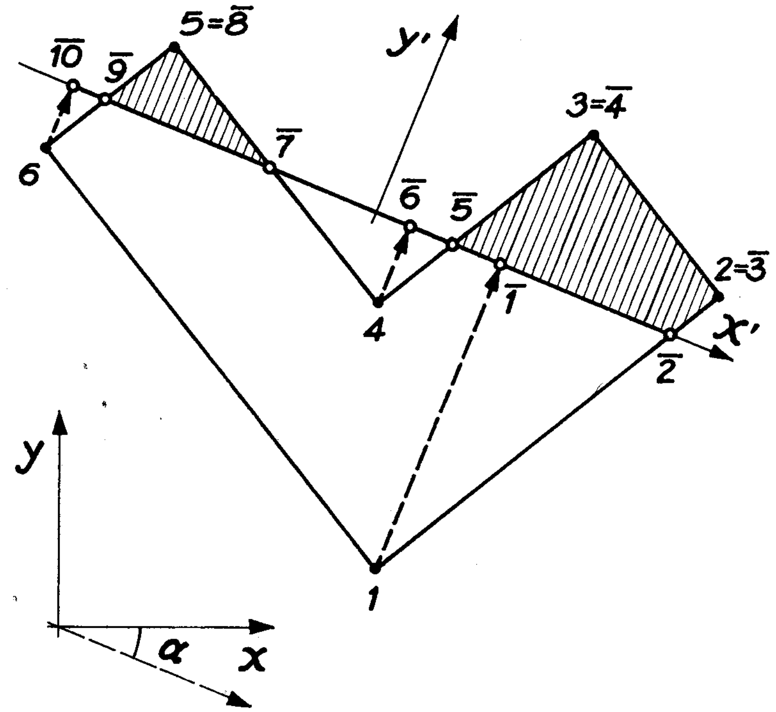

FIG. 2.—EXAMPLE OF SUBFIGURE.

Therefore, it is apparent that by such modification of an arbitrary closed simply connected contour, it is possible to get a variable number of different independent closed figures

which ought to be determined in the same form as the original one so that the previous numerical technique might be applied. An extension of the above is the case when the subfigure has to be limited by two straight lines parallel to the divisor direction, Fig. 3; what has been called a band subfigure here.

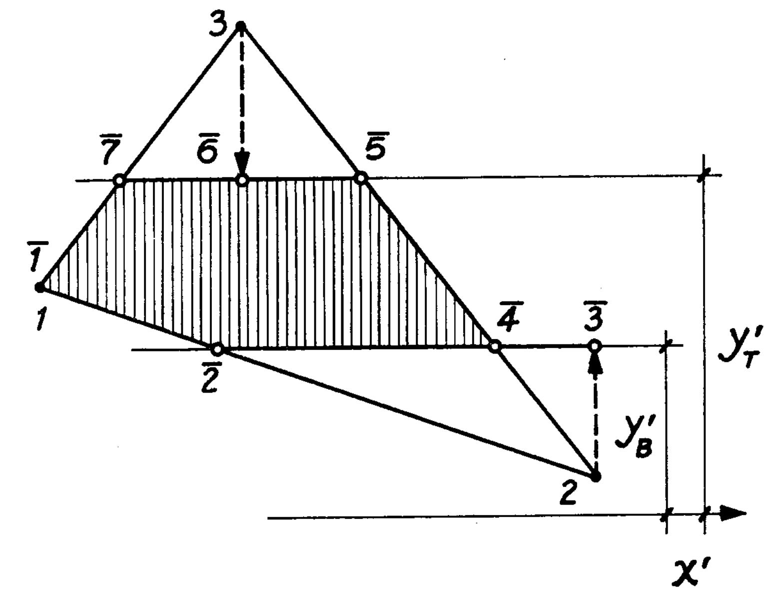

FIG. 3.—BAND SUBFIGURE AND ITS RESULTING BAND POLYGON

The algorithm, whose logic diagram is presented in Fig. 4, details the general procedure developed by the author to solve this problem as a preliminary step to the computations. The idea is very simple and is illustrated in Figs. 2 and 3.

A new unique closed polygon is formed with all the possible parts of the subfigure in such a way that, maintaining the selected sense of numbering, the vertices outside the band are projected on its limiting lines and as many new points are introduced in it as there are intersections between the original contour and the parallel limiting straight lines.

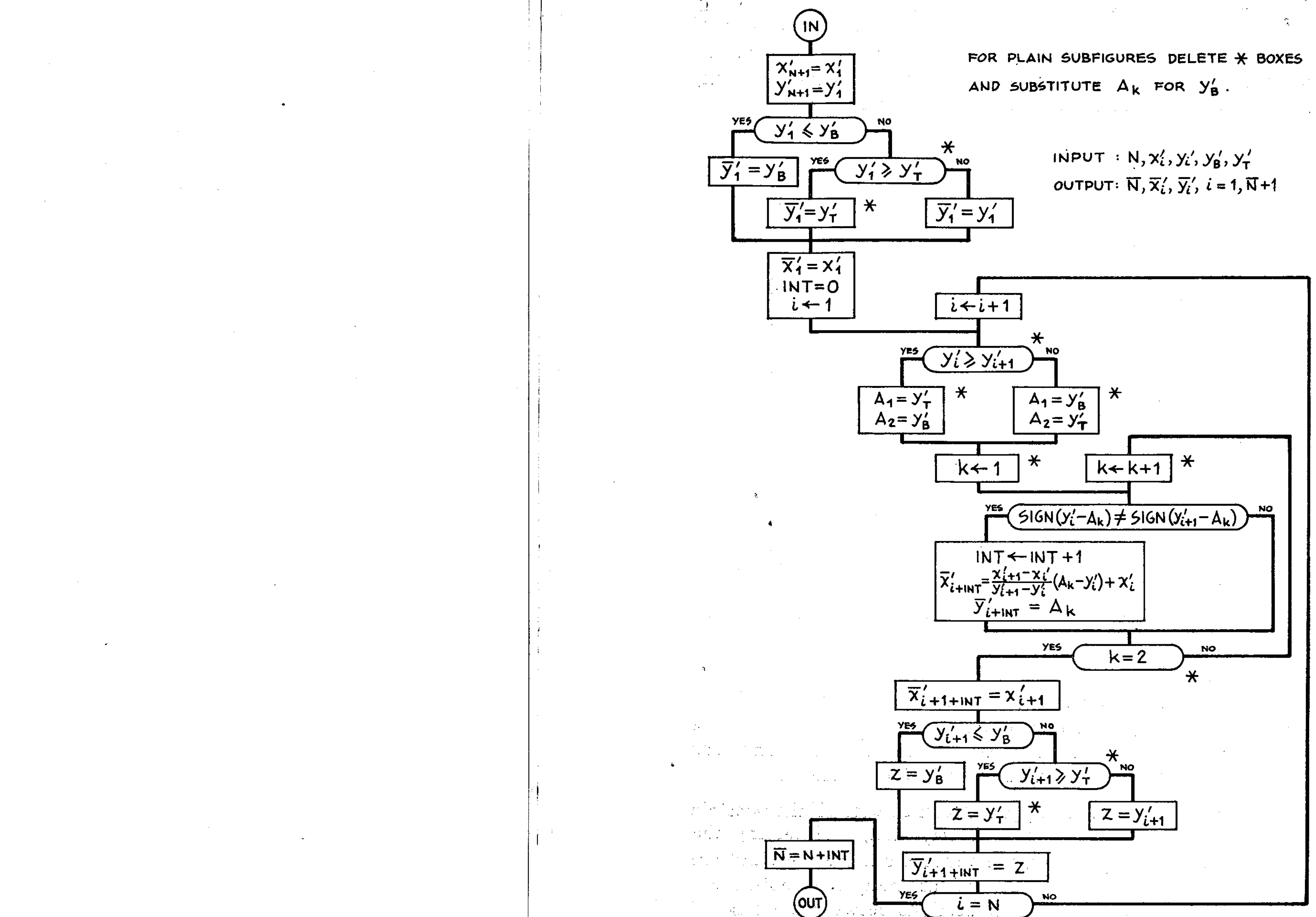

FIG. 4.—THE BAND POLYGON ALGORITHM

This short algorithm allows an easy unified approach to all shapes. The generation of such a unique new band polygon avoids a tremendous programming effort in determining how many different zones might appear.

If the upper limiting parallel line passes through the highest vertex as in plain subfigures, the algorithm is further simplified. Then A_k may be substituted by y'_B and boxes denoted by an asterisk in Fig. 4 may be eliminated.

In structural engineering plain subfigures are basic in the study of the ultimate strength of reinforced concrete cross sections subjected to combined axial load and biaxial bending using the rectangular stress block. In the revision option their areas and centroids must be computed hundreds of times. This algorithm made possible the general program quoted in Reference 2, which directed to shear wall cross sections actually motivated the whole approach. This program was successfully applied in practice in designing the plans of the 43 story type structure of the "Parque Central" building complex in Caracas, Ref. 5, defined by 64 vertices and 46 bar concentrations, through the computation of their interaction volumes and constant load contours.

On the other hand, band subfigures are essential if any other stress distribution is to be considered and if strains different from those at ultimate must be investigated. For computing the buckling capacity of members the band polygon algorithm and the formulae presented provide a general, more expedient and approximate method that those known to the author, since any bending direction or shape can be directly handled with the definition of a set of strips parrallel to the neutral axis. In contrast with other grid methods, Refs. 4 and 6, this builds the specific band geometric division needed to solve ordinary bending strain problems, i.e., where the stress magnitude varies only with the distance of the band to the neutral axis. In fact, the analytical approximation depends on one variable only, the number of strips into which the user decides to divide the compressed zone. Furthermore, since this band division can be performed for each compressed zone without significant computer cost, it is possible then to get a uniform precision independent on the depth of the neutral axis, Ref. 3.

5. CONCLUSIONS

1. The method and formulae presented provide a simple, systematic and exact numerical technique for computing mechanical characteristics of plane figures formed by simple connected polygonal contours referred to a pair of coördinate axes, arbitrarily selected by the user.

2. Since multiple connected contours can be transformed into simply connected ones by proper fictitious sectioning and curved contours may be conveniently approximated by polygons, this numerical technique is applicable to all kinds of plane figures in engineering practice.

3. The simplicity of the formulae and the direct reference of results to the axes of interest, eliminating translations and rotations, allow that the most involved figure can easily be computed either by straightforward tabulation or automatically by a small electronic desk computer.

4. The proposed subfigure polygon algorithm provides a simple general subprogram for calculating mechanical properties of zones of any plane figure generated by parallel straight lines, as they are required to be considered in strength of materials when studying the combined axial load and biaxial bending or buckling member capacities. In addition, subfigure polygons find application in plotter programming when large drawings are to be divided and their parts adjusted to form sizes.

5. The author believes that this numerical technique which solves such a frequent engineering problem is extremely expedient and useful to apply. Due to its wide practical importance there is no justification in keeping it away from students and engineering handbooks as it appears to have been the case until now. This might explain why a general accurate program for determining the ultimate strength of arbitrary

reinforced concrete cross sections has not been yet reported, under conditions of combined axial load and skew bending, with the exception given by Reference 2.

Remark: This paper is an appendix to the author's doctoral thesis.

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